Briefing #2 - L-Band Channel Modeling

Future Communications Study
Phase II End of Task Briefing

June 21, 2006
Structure and Content of the Presentations

Stake Holder Needs
- Compatibility
- Affordability
- Availability

Briefing 2: L-Band Propagation Model
Briefing 3: L-Band Modeling & Analysis
Briefing 4: L-Band Business Case
Briefing 7, 11 & 12: SATCOM Studies

Briefing 5: Evaluation Criteria Development
Briefing 6: Evaluation Process Details
Briefing 8: Technology Evaluation Results
Briefing 7 & 10: C-Band Studies
Briefing 9: Future Work

Input Candidate Technologies

Consensus Documentation

Evaluation Criteria

Technology Screening

Rec. for C-Band
Rec. for Sat
Rec. for L-Band

Detailed Evaluations
Briefing Outline

• Study Objective and Motivation
• Channel Modeling Background
• Study Approach
• Study Results
• Conclusions
Study Objective and Motivation

• What we are doing
  – We are developing small-scale propagation models to characterize the Aeronautical Air/Ground Channel in L-Band
    • Small-scale models are essential in simulating communications system performance
    • Models will be used to estimate candidate Future Radio System (FRS) performance

• Why we are doing this
  – After an extensive literature search we concluded that little work has been done in L-band for Air/Ground communications
  – While measurements exist for terrestrial channels, no measurements currently exist for the Air/Ground channel
  – An understanding of the statistical variations of the propagation environment is fundamental to optimizing communication system performance

• What is the expected task output
  – We expect to develop representations of the L-Band aeronautical air/ground channel that characterize the fading behavior of the channel and can be used in waveform simulations of FRS candidates
Channel Modeling
Background

• Propagation models are typically classified as either Large Scale Propagation Models or Small Scale Fading Models

<table>
<thead>
<tr>
<th>Large Scale Propagation Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagation models that predict the mean signal strength for an arbitrary transmitter-receiver separation distance to facilitate estimation of radio coverage area and are referred to as Large Scale Propagation Models</td>
</tr>
<tr>
<td>Characterized by a slow change in average received power with increasing distance from the transmitter. To get a sense of average received power, measurements are averaged in a local area over 10’s of wavelengths</td>
</tr>
<tr>
<td>These models are useful for link budgets and coverage analysis</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Small Scale Fading Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagation models that characterize the rapid fluctuations of the received signal strength over very short distances or short time durations are referred to as Small Scale Fading Models</td>
</tr>
<tr>
<td>Characterized by rapid and severe changes in received signal amplitude (several orders of magnitude) with motion over very short distances.</td>
</tr>
<tr>
<td>These models are essential for proper waveform design and optimizing receiver implementation</td>
</tr>
</tbody>
</table>

Small Scale Fading Models are the focus of this modeling effort.
• Small scale fading models can be classified as “frequency-selective” or “frequency-nonselective” (also called flat) fading models
  – Both flat and frequency-selective fading degrade system performance
  – Frequency-selective fading channels result in an irreducible BER
    • Mitigated by adaptive equalization, spread spectrum techniques, OFDM or insertion of pilot signals
  – Flat fading can result in destructive interference, due to the phase differences in the unresolvable multipath components
    • Mitigated by diversity and error-correction coding
  – Simply put, while large-scale models help us predict $E_b/N_0$, it is the channel fading characteristics that determine system performance

Error Performance
Bernard Skylar, “Rayleigh Fading Channels in Mobile Digital Communications Systems Part II: Mitigation”

Irreducible BER
Justin Chuang, “The Effects of Time Delay Spread on Portable Radio Communications Channels with Digital Modulation”
After an extensive literature search we concluded that very little measured data exists to characterize the fading behavior of the L-band Air/Ground communications channel.

In order to have a useful model for waveform simulation and evaluation of candidate Future Radio System technologies, the following additional elements need to be estimated:

- Delay Spread
- Doppler Power Spectrum
- Tap amplitudes, # of taps, fading processes, and correlation between taps

While no measurements exist that could be used to infer these quantities directly, there is sufficient theory and analogy to be made to the body of land mobile measurements to provide a basis for estimation.

The next section provides the details of our process to estimate the delay spread and the Doppler power spectrum parameters. The number of taps to be used in a simulation is technology dependent (given a derived excess delay spread).
Study Approach

- In order to form estimates of the delay spread and delay spread statistics, a ray-tracing simulation was developed.
- The ray-tracing simulation models both diffuse and specular reflections from the Earth’s surface.
- Many terrain models could have been selected for this study.
- Our initial approach used a flat terrain model, but after our initial investigation we concluded that mountainous terrain provides a worst-case scenario.
Study Approach – Selection of Simulation Topography

- Mountainous terrain, in the en-route case, has the potential to provide extremely long multipath delays.
- Long delay spreads either limit the data rate that can be transmitted or require special techniques to achieve required performance.
- In an effort to characterize a worst case scenario for multipath delay spread, we selected Aspen, CO.
  - Aspen terrain and the current RCAG site are shown in the picture.

RCAG site
Study Approach – Simulation Context

- Simulation context is shown in the picture
  - Uses bi-static radar equation and published ranges for normalized cross section in a Monte-Carlo simulation framework
- Simulation uses both Monte-Carlo and ray tracing techniques
  - Monte-Carlo elements include randomly selected aircraft position & heading (ground station is fixed) and radar cross sections
  - Ray tracing is used to calculate requisite distances
  - To identify unique multipath components, a method of concentric oblate spheroids is employed
- Although the diagram illustrates only the diffuse, both specular and diffuse multipath components are considered
Study Approach – Simulation Flow Chart

• Flow Chart for overall simulation:

- Import Terrain Data
- Define a Coordinate System
- Transform Terrain Data Format (Lat, Long, El) → (X,Y,Z)
- Assign the Rx Location
- Pick a Tx Location (Aircraft location) (Uniform X, Y, Quantized Z)
- Step through terrain model and calculate multipath dispersion using a method of concentric oblate spheroids
- Generate PDP
- Reiterate with another Tx Location?
  - Yes
  - No
- Summarize Statistics
Study Approach – Import & Format Terrain Data

- The U.S. Geological Survey (USGS) offers free downloads from their National Elevation Dataset (NED)
  - The NED is a seamless raster product derived primarily from USGS 30-meter Digital Elevation Models (DEMs)
  - Terrain data with finer resolution is available, but would be cumbersome to use as increased resolution increases simulation runtime
- The terrain elevation data is reformatted so that the origin (x=0, y=0) is located at the southwest corner
- The tower and aircraft locations are defined and the Power Delay Profile (PDP) is generated for the channel using ray-tracing techniques
  - The location of the RCAG site is shown in the figure
Study Approach – Method of Concentric Oblate Spheroids

• The method used for generating the PDP is as follows:
  – A series of concentric oblate spheroids is generated using the Tx & Rx locations as the focal points
    • The semi-minor axis for each successive spheroid is increased by a fixed increment
  – The contour of terrain trapped between two successive spheroids is used to calculate multipath dispersion for a particular time delay
  – Each contour consists of a set of terrain points that take the shape of a distorted annulus
  – These points undergo a ray-tracing analysis (example contours are shown to the right)
Study Approach – Terrain Analysis Flow Chart

- Flow Chart for analyzing terrain model:

  - For each iteration, for each terrain elevation point...
  - Is point LOS from both Tx & Rx? [Y/N]
  - Calculate EI And Az from Tx & Rx in the plane defined by the terrain at that point
  - Eliminate NLOS pts
  - Are conditions for specular reflections met? [Y/N]
  - Point contributes to multipath as a specular reflection
  - Calculate reflection coefficient, $\rho$, from the geometry of the Terrain (Elevation $\triangle$)
  - Calculate magnitude and phase of components from $\rho$ and free space path loss
  - Point contributes to multipath as a scattered reflection
  - Cluster points to form scattering surfaces
  - Assign normalized scattering cross-section, $\sigma^0$, based on elevation & tree-line
  - Calc. magnitude of components via bistatic radar eqn & assign phase from uniform dist.
  - Sum multipath components of all types for total Rx power per iteration
A LOS algorithm was needed to determine points on the terrain that are in view of both the transmitter and the receiver. These points provide potential for multipath reflections.

An internet search turned up a paper: “Line-of-Sight Attributes for a Generalized Application Program Interface” by Michael D. Proctor and William J. Gerber. This paper outlines several different approaches for determining LOS visibility with varying levels of fidelity and computational complexity.

The “four-post interpolation” method outlined in this paper was implemented

- The four-post interpolation method is the most computationally complex of the LOS algorithms listed, but also provides the highest fidelity.
Study Approach – LOS Methodology (2)

• Four-Post Interpolation Method
  – Given a transmitter location and the terrain elevation data, determine whether each terrain elevation data point is in view
  – For each terrain elevation data point, create a three-dimensional (3D) ray from that point to the Tx.
  – Step through the 3D ray (at quantized regular intervals) and compare the elevation of the ray with the elevation of the underlying terrain. If the point on the ray does not fall on the grid, interpolate using the four surrounding elevation data points.

Study Approach – LOS Methodology (3)

- Example LOS results

<table>
<thead>
<tr>
<th>I. Tx</th>
<th>II. Rx</th>
<th>III. Tx ∩ Rx</th>
</tr>
</thead>
</table>

(LOS = red, NLOS = blue)

I. The Tx/Aircraft has very high altitude and therefore can see much of the underlying terrain

II. The Rx/Tower is relatively close to the ground and has less visibility of the terrain

III. This figure represents the intersection of the Tx and Rx LOS matrices
Study Approach – Specular or Diffuse Reflections

- Points on the terrain that are in view of both the Tx & Rx will contribute to multipath dispersion as either specular or diffuse reflections

- The conditions for specular reflections are much more stringent than those for diffuse reflections
  - If a point satisfies those conditions, that point will contribute to multipath as a specular reflection
  - All points that do not satisfy these conditions will contribute to multipath as part of a scattering surface
    - Note that this is a conservative assumption & overestimates multipath
Study Approach – Specular Reflections

- Process for determining Specular Reflectivity
  - For a particular point of interest (terrain data point):
    - Select two adjacent points on the terrain to form a plane and solve for the equation of that plane
    - Calculate the elevation and azimuthal angles of the incident ray in that plane
    - Calculate the elevation and azimuthal angles as seen by the receiver
    - Two conditions must be met for specular reflectivity:
      1. \( E_{\text{inc}} = E_{\text{ref}} + \Delta_{\text{tolerance}} \)
      2. \( A_{\text{inc}} = A_{\text{ref}} + 180^\circ + \Delta_{\text{tolerance}} \)
If a point satisfies the conditions for specular reflectivity, the power and phase of the specular component must be calculated:

- The power is calculated using the free-space path loss model in conjunction with the reflection coefficient $|\rho_v|$
- The phase is a function of the distance traveled, the frequency, and the phase change due to reflection, $\angle \rho_v$
- The equation for $\rho_v$ is:

$$\rho_v = \frac{(\varepsilon_r - jx)\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}}{(\varepsilon_r - jx)\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}}$$
Study Approach – Scattered Reflections: Bistatic Radar Eqn

• All points that do not meet conditions for specular reflectivity contribute as scattering surfaces
  – A majority of points contribute to multipath in this fashion
• The bistatic radar equation is used to estimate the power of the resulting multipath components due to scattering

\[
L(t_d) = \sum_k \frac{\lambda^2}{(4\pi)^3} \cdot \frac{\sigma_0^0 dA_k}{r_{TS_k}^2 r_{SR_k}^2 r_{TR_k}^2}
\]

where,
• \(\sigma^0\) is the normalized scattering cross-section of the patch of area \(dA_k\)
• \(dA_k\) is the Area of the \(k^{th}\) patch of area \(S_k\)
• \(r_{TS}\) is the distance from the Tx and the reflective surface
• \(r_{SR}\) is the distance between the reflective surface and the Rx
• \(r_{TR}\) is the distance between the Tx and the Rx
• \(L(0)\) is the free space path loss (Friis) from the Tx to the Rx
• \(k\) is the quantized terrain data
To properly apply the bistatic radar equation, a good estimate for the normalized scattering cross-section, $\sigma^0$, must be used

- $\sigma^0$ is determined by the landscape of the scattering region

The terrain throughout the Aspen, CO region consists of both tree-covered areas as well as bare, rocky slopes.

An “alpine tree-line” exists in the rocky mountains defined as the highest elevation at which trees are typically found on mountains; higher up, it is too cold and windy to sustain vegetation.
The tree-line infers a model for choosing the appropriate $\sigma^0$:

**Heterogeneous Model**

- If the point of intersection is above the tree line, choose $\sigma^0_A$ to be between (-7 and -10 dB)*
- If the point of intersection is below the tree line, choose $\sigma^0_B$ to be around -21 dB*

* Values from measured data (see Dreissen)

Using this heterogeneous model for the tree-line, the points are separated into two groups:

1. Those above the tree-line ($\sigma^0 = -7$ dB)
2. Those below the tree-line ($\sigma^0 = -21$ dB)
Study Approach – Scattered Reflections: $dA_k$

• To properly apply the bistatic equation, the area of the region, $dA_k$, must also be properly estimated
  – A rough estimate of $dA_k$ is sufficient as a 50% error in area estimation results in only a 3 dB change in Power

• The values for $\sigma^0$ found in the literature have been normalized to areas on the order of 1 km$^2 = 10^6$ m$^2$

• A single point of terrain data is representative of regions on the order of 22,500 m$^2$ (using a simulation scaling factor = 5)

• Since $10^6$ m$^2 >> 22,500$ m$^2$, points must be clustered together to form scattering regions on the order of $10^6$ m$^2$
The nearest neighbor, agglomerative hierarchical clustering algorithm is used to cluster points together to form scattering surfaces whose size is on the order of 1 km\(^2\).

The algorithm is an iterative algorithm that works as follows:

- Create a distance matrix that includes the distance from every point to every other point in the set.
- Find the minimum distance in the matrix.
  - Cluster the points associated with the minimum distance.
  - Remove the rows and columns of the clustered points.
- Repeat this process until all rows and columns are accounted for.
  - Note that starting with an odd number of points results in a “leftover” point.
- During the next iteration, the distance matrix is regenerated and the two clusters that are closest in distance are joined.
- This process continues until the desired number of clusters is obtained.

After the clustering operation is completed the magnitude of each scattered multipath component is calculated via the bistatic radar equation.

The phase of a scattered multipath component is randomly chosen from a uniform distribution from \([0…2\pi)\)
Study Approach – Combining Specular and Diffuse Multipath

- The method for generating PDPs previously described results in three types of multipath components per at each time delay:
  1. Specular Reflections
  2. Diffuse Reflections from above the tree-line
  3. Diffuse Reflections from below the tree-line

- Any, all, or none of these multipath components might exist for a particular time delay

- These multipath components are vectorally added and the resultant sum represents the power received for that delay

- The resultant PDP of the channel consists of a time-delay vector and an accompanying received power vector
Study Results – Typical Simulation Outputs

- The L-Band Channel Estimator Simulation has generated hundreds of Power Delay Profiles (PDPs)

- Data reduction techniques must be employed in order to extrapolate channel model parameters from the PDPs
Study Results – PDP Re-Binning

• The PDPs generated by the simulation contain multipath delays that are not uniformly spaced
• The non-uniform spacing can be attributed to the geometry of the concentric oblate spheroids
  – The semi-minor axis of each successive oblate spheroid is increased by a fixed increment resulting in a non-linear spacing of multipath delays
• The PDP multipath powers must be re-centered in accordance with the sampling rate of the FRS simulation
• Multipath components that exist between the desired spacing are shifted to adjacent components before and after the current position
• The amount of power that goes to the new spacing is proportional to the distance from the desired spacing

Proportionally more power would go to the component on the right because it is closer to the current position
Study Results – Post-Processing of Simulation Outputs

• After re-binning the PDPs, the next step in the data reduction process is determining the Minimum Validity Threshold
  – The PDPs generated by the L-Band Channel Estimator Simulation contain multipath components that range from just a few dB to tens of dB down from the LOS component
  – If these were true measurements, many of the multipath components would not be distinguishable from the noise floor of the measurement equipment
  – The simulation differs from measurements in that it does not have a noise floor
    • For some PDPs that consist solely of very low-power multipath returns, a skewing of delay spread statistics is observed in the model
    • This behavior, while perhaps real, is not likely to be significant due to the nature of our channel (Rician)
    • In other words, although they show up in the model, these low-power returns would not degrade system performance given the presence of a strong LOS component
  – A threshold level termed the Minimum Validity Threshold (MVT) was defined to eliminate very-low power multipath returns
Study Results – Post-Processing of Simulation Outputs (2)

• Methodology for Determining the MVT
  – Start with a range of values for MVT (literature suggests 20-25 dB [Matolak])
  – Plot the relative frequency (pdf) of the RMS delay spread after applying a range of MVT values (i.e. – 20, 21, …, 25 dB) to the PDPs
  – Calculate the RMS delay spread for using each MVT
  – These pdfs are fitted to known distributions so that the statistics of the distributions represent the statistics of the channel for a particular MVT
  – Literature suggests that the pdf of RMS delay spread for a Rician channel is exponential
  – A best fit is performed for each pdf and the residual error is calculated
  – The pdfs are similar to one another, so the pdf with the least residual error (best fit) is selected
• Defining the $MVT = 22$ dB infers a model from which we can calculate delay spread statistics.
Study Results – Suggested Channel Models

• After applying the MVT to all of the PDPs, the mean RMS-DS was calculated to be 1.4 µs

• It is instructive to consider representative technologies at this point as the technology data rate will drive model parameter estimation
  
  – A rule of thumb that is frequently applied is if the mean RMS-DS is at least one tenth of the symbol duration, then the channel is frequency selective (Rappaport 170)

  – Flat models differ in structure from frequency-selective models. Required simulation sampling rates also have an impact on channel model structure
Study Results –
Suggested Channel Models

• In order to illustrate this, two technologies that scored well in the FCS Pre-Screening were selected for analysis: P34 and LDL

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Data Rate</th>
<th>Symbol Duration</th>
<th>1/10th of the Symbol Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDL</td>
<td>62.5 kbps</td>
<td>16 µs</td>
<td>1.6 µs</td>
</tr>
<tr>
<td>P34</td>
<td>4.8 ksps*</td>
<td>208.3 µs</td>
<td>20.83 µs</td>
</tr>
</tbody>
</table>

* P34 is an OFDM system. The tabulated data rate is per carrier and is the symbol rate. Overall P34 data rates range from 76.8-691.2 kbps

• Given our simulated channel mean RMS-DS,
  – P34 should undergo flat fading
  – LDL presents a borderline case because the mean RMS-DS is very close to one tenth of the symbol duration

• For this reason we have decided to develop a frequency-nonselective fading model for P34 and a frequency-selective fading model for LDL
Study Results – Suggested LDL Channel Model

- Channel Model for LDL:
  - A deterministic simulation model for a frequency-selective mobile radio channel (Pätzold 270):

- The parameters that define the LDL channel model are:
  - # of Taps (N)
  - Tap Spacing \( (D_1, D_2, \ldots, D_N) \)
  - Tap Weights \( (a_0, a_1, \ldots, a_N) \)
  - Tap Fading Processes \( (\mu_0, \mu_1, \ldots, \mu_N) \)
  - Other considerations:
    - Correlation between Taps
Study Results – Suggested LDL Channel Model

• Deriving the # of Taps
  – Each of the simulated PDPs contained a large number of multipath components
    • Some are more prominent than others on average
    • A good model would emulate the simulated channel without undue complexity
      – Should require the minimum number of taps required to achieve a “good fit”
      – Many researchers [Matolak] use the contribution of a tap to total energy as a barometer of which taps are required
      – Using this method, one selects the number of taps required to account for X% of total PDP energy
        » We have selected X = 99% for our threshold
    – Plotting the cumulative energy per tap shows that 99% of the energy appears within the first 7 taps

• The equation for cumulative energy through the jth tap across j PDPs is:

\[
CumulativeEnergy_i = \frac{\sum_{j=1}^{i} TapEnergy_j}{TotalCumulativeEnergy}
\]
• Tap Spacing
  – The tap delays coincide with the sampling rate of the simulation they will be used in

• Such simulations require a sampling (typically over-sampling) rate that is an integer multiple of the symbol rate

• Aliasing concerns drive typical sampling rates to be on the order of 10 samples per symbol

– Hence for LDL the tap spacing, \( t_0 = 1.6 \, \mu s \) (LDL symbol duration is 16 \( \mu s \))
• Tap Weights
  
  – A plot of the average energy per tap shows the mean amplitude for each tap

  ![Average Energy per Tap Diagram]

• The equation for average energy for the $i^{th}$ tap across $j$ PDPs is:

\[
\text{AverageEnergy}_i = \frac{\sum_j \text{TapEnergy}'_j \times \text{TapStatus}'_j}{\sum_j \text{TapStatus}'_j}
\]
Study Results – Suggested LDL Channel Model

• Tap Fading Processes
  – Pdf’s for each tap (#’s 1→7) were fit to known distributions with minimal RMS error so that the fading processes could be modeled
  – The table below lists the fading process, statistical mean, and variance for each of the taps:

<table>
<thead>
<tr>
<th>Tap #</th>
<th>Delay (µs)</th>
<th>Power (lin)</th>
<th>Power (dB)</th>
<th>Fading Process</th>
<th>Doppler Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Ricean</td>
<td>Jakes</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>0.0359</td>
<td>-14.5</td>
<td>Rayleigh</td>
<td>Jakes</td>
</tr>
<tr>
<td>3</td>
<td>3.2</td>
<td>0.0451</td>
<td>-13.5</td>
<td>Rayleigh</td>
<td>Jakes</td>
</tr>
<tr>
<td>4</td>
<td>4.8</td>
<td>0.0689</td>
<td>-11.6</td>
<td>Rayleigh</td>
<td>Jakes</td>
</tr>
<tr>
<td>5</td>
<td>6.4</td>
<td>0.0815</td>
<td>-10.9</td>
<td>Rayleigh</td>
<td>Jakes</td>
</tr>
<tr>
<td>6</td>
<td>8.0</td>
<td>0.0594</td>
<td>-12.2</td>
<td>Rayleigh</td>
<td>Jakes</td>
</tr>
<tr>
<td>7</td>
<td>9.6</td>
<td>0.0766</td>
<td>-11.2</td>
<td>Rayleigh</td>
<td>Jakes</td>
</tr>
</tbody>
</table>
Study Results – Suggested P34 Channel Model

• Channel Model for P34:
  – The P34 channel model is less complex than the LDL channel model because the channel is frequency-nonselective and has the form:

\[ x(n t_0) \quad \Rightarrow \quad y(n t_0) \]

  – The Ricean fading process is derived in the complex baseband by creating two colored Gaussian processes
    • Rice method used to generate Gaussian Process (summation of sinusoids whose coefficients and frequencies are determined by the Doppler Power Spectrum of the channel)
    – As the process is Ricean, a time-variant mean is summed with the colored Gaussian random process
    – The magnitude of the complex-enveloped Gaussian colored processes yields the Ricean process with fade durations and amplitudes determined by the channel
Conclusions

- Conclusions
  - An RMS delay spread of 1.4 µs was predicted for a certain distance (average distance = 40 miles) from the transmitter in mountainous terrain
  - A generalized model, using methodology of Greenstein, Erceg, Yeh, & Clark, can be used to extend our model to any separation distance and has the form:

\[
\overline{\sigma}_\tau = \overline{\sigma}_{\tau_0} d^\varepsilon A
\]

- where,
  - \(d\) is the distance in km
  - \(\overline{\sigma}_{\tau_0}\) is the median value of the RMS delay spread at \(d = 1\) km
  - \(\varepsilon\) is an exponent that lies between 0.5-1.0, based on the terrain type
  - \(A\) is a lognormal variate
Conclusions (2)

• RMS delay spreads ($\tau_{RMS}$) were predicted for reference distances of 1 km and 64.37 km (40 miles)
  - $\tau_{RMS}(1 \text{ km}) = 0.1 \mu$s
  - $\tau_{RMS}(64.37 \text{ km}) = 1.4 \mu$s

• The upper bound for a single sector is on the order of $\sim$320 km (200 miles), making the maximum aircraft-tower separation distance of $\sim$160 km (100 miles)

• Fitting the Greenstein model to the reference data allows an estimate of $\tau_{RMS}$ for a 160 km (100 miles) aircraft-tower separation distance
  - Using $\epsilon = 0.6337$ and $A = 6 \text{ dB}$, $\tau_{RMS}(160 \text{ km}) = 2.5 \mu$s